Branch – Mechanical engg.
Semester – 4th
Sub: Fluid Mechanics and Machinery (1625405)

# UNIT – 04

## Flow through pipes

- **Reynold's Number – (Re)**

$$Re = \frac{\text{Inertia force}}{\text{viscous force}} = \frac{\rho v d}{\mu}$$

where, $\rho$ = density
$v$ = avg. velocity
$d$ = diameter of pipe
$\mu$ = viscosity of fluid

It is a dimensionless quantity

- **Nature of flow a/c to Reynold's Number for pipe and open channel flow:** –

The limiting values of Reynold's number corresponding to which flow is laminar is given by –

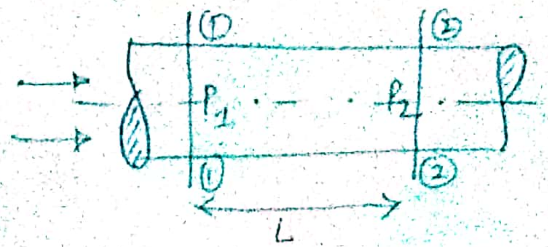
| Flow condition | Pipe flow | Open channel flow |
|---|---|---|
| Laminar flow | $Re \leq 2000$ | $Re \leq 500$ |
| Transitional flow | $2000 \leq Re \leq 4000$ | $500 < Re < 1000$ |
| Turbulent flow | $Re > 4000$ | $Re > 1000$. |

- Flow is viscous or laminar at very low velocity. At low velocity fluid moves in layers. Each layer of fluid slides over the adjacent layer. Due to relative velocity between the two layers the velocity gradient $\frac{du}{dy}$ exists and hence a shear stress $\tau = \mu \frac{du}{dy}$ acts on the layers.

- **Loss of pressure head for a given length (L) of a pipe in laminar flow.**

Loss of pressure head $= \frac{P_1 - P_2}{\rho g}$

$$\boxed{\frac{P_1 - P_2}{\rho g} = h_f = \frac{32 \mu \bar{u} L}{\rho g D^2}}$$



$\bar{u}$ = avg. velocity

This equation is called Hagen Poiseuille formula.

- **Turbulent flow :—**

· Laminar flow is possible only at low velocities and when the fluid is highly viscous. But when the velocity is increased or fluid is less viscous, the fluid particles do not move in straight lines.

· The fluid particles move in random manner resulting in general mixing of the particles. This type of flow is called turbulent flow.

- **Frictional loss in pipe flow :—**

When a liquid is flowing through a pipe, the velocity of the liquid layer adjacent to the pipe wall is zero. The velocity of liquid goes on increasing from the wall and thus velocity gradient and hence shear stresses are produced in the whole liquid due to viscosity. This viscous action causes loss of energy which is usually known as frictional loss.

On the basis of his experiments, William Froude gave the following laws of fluid friction for turbulent flow :—

The frictional resistance for turbulent flow is —

(i) proportional to $V^n$, where n varies from 1.5 to 2.0,
(ii) proportional to density of fluid
(iii) proportional to area of the surface in contact,
(iv) independent of pressure,
(v) dependent on the nature of the surface in contact.

# Loss of energy in pipes

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost.

## Energy losses

**1. Major energy losses**

This is due to friction and it is calculated by the following formulae:

(a) Darcy-Weisbach formula
(b) Chezy's formula

**2. Minor energy losses**

This is due to :-

(a) Sudden expansion of pipe
(b) Sudden contraction of pipe
(c) Bend in pipe
(d) pipe fittings etc.
(e) obstruction in pipe

## * Darcy Weisbach formula :- (Loss of head due to friction)

$$\boxed{\text{loss of head } (h_f) = \frac{4fLv^2}{2dg}}$$

where, $f$ = co-efficient of friction
$v$ = avg. velocity
$L$ = length b/w section 1 & 2

'f' is a function of Reynold's Number.

So, $f = \frac{16}{Re}$ for $Re < 2000$ (laminar flow)

$= \frac{0.079}{Re^{1/4}}$ for $(4000 < Re < 10^6)$, Turbulent flow.

Somewhere, the formula for loss of head is written in the form of friction factor.

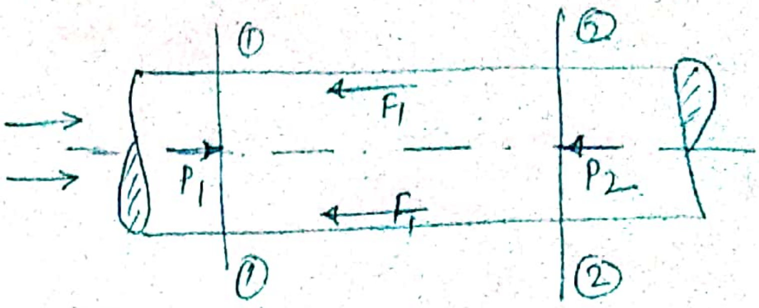
$$\boxed{\text{loss of head } (h_f) = \frac{fLv^2}{2dg}}$$

Here, $f$ = friction factor
$= \frac{64}{Re}$ for laminar flow.

Note : See derivation of Darcy weisbach formula in R.K. Bansal book chapter : 10.

## * Chezy's formula for loss of head due to friction in pipes :-

Applying Bernoulli's equations b/w sections 1-1 and 2-2.




Total head at ① = (Total head at 2) + (loss of head due to friction b/w ① & ②)

$$\frac{p_1}{\rho g} + \cancel{\frac{v_1^2}{2g}} + \cancel{z_1} = \frac{p_2}{\rho g} + \cancel{\frac{v_2^2}{2g}} + \cancel{z_2} + h_f$$

$z_1 = z_2$ as pipe is horizontal
$v_1 = v_2$ as dia of pipe is same at sections ① & ②

$$\Rightarrow \frac{p_1}{\rho g} = \frac{p_2}{\rho g} + h_f \Rightarrow h_f = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} \quad \text{—(I)}$$

Now frictional resistance = (frictional resistance per unit wetted area per unit velocity) × (Wetted area) × (velocity)$^2$

$$F_1 = f' \times \pi d L \times v^2 \qquad [\because \text{wetted area} = \pi d \times L, \; V = V_1 = V_2]$$

$$= f' \times P \times L \times v^2 \quad \text{—(II)} \qquad [\because \pi d = \text{Perimeter} = P]$$

Now resolving all the forces in the horizontal direction,

$$p_1 A - p_2 A - F_1 = 0$$

$$(p_1 - p_2) A = F_1 = f' \times P \times L \times v^2$$

$$p_1 - p_2 = \frac{f' P L v^2}{A}$$

But from eqn (I), $p_1 - p_2 = \rho g h_f$

$$\therefore \rho g h_f = \frac{f' P L v^2}{A}$$

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L v^2 \quad \text{—— (III)}$$

Now, the ratio $\frac{A}{P} = \frac{\text{Area of flow}}{\text{Wetted perimeter}}$ is called hydraulic mean depth or hydraulic radius and is denoted by $m$.

Hydraulic mean depth (m) $= \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$

Substituting $\frac{A}{P} = m$ or $\frac{P}{A} = \frac{1}{m}$ in eqn (III)

$$hf = \frac{f'}{\rho g} L v^2 \times \frac{1}{m} \quad \text{or} \quad v^2 = hf \times \frac{\rho g}{f'} \times m \times \frac{1}{L} = \frac{\rho g}{f'} m \times \frac{hf}{L}$$

$$V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{hf}{L}} = \sqrt{\frac{\rho g}{f'}} \sqrt{m \frac{hf}{L}}$$

Let $\sqrt{\frac{\rho g}{f'}} = C$, where, $C$ = chezy's constant

and $\frac{h_f}{L} = i$, where, $i$ = loss of head per unit length of pipe,

then we get. $\boxed{V = C\sqrt{mi}}$ —— (IV)

chezy's formula.

- Loss of head due to friction in pipe from chezy's formulae can be obtained if the velocity of flow through pipe and also 'C' is known.

Ex. Find the head lost due to friction in a pipe of diameter 300 mm and length 50m, through which water is flowing at a velocity of 3 m/s using- (I) Darcy formula (II) chezy's formula. given $\nu = 0.01$ stoke for water.

Sol^n Given, $d = 300\text{mm} = 0.3\text{m}$

length of pipe $(L) = 50\text{ m}$.

velocity of flow $(V) = 3\text{ m/s}$

chezy's constant $(C) = 60$

Kinematic viscosity $(\nu) = 0.01$ stoke $= 0.01\ \text{cm}^2/\text{s}$
$= 0.01 \times 10^{-4}\ \text{m}^2/\text{s}$

(I) Darcy formula

$$h_f = \frac{4fLv^2}{2dg}$$

where, $f$ = coefficient of friction, $Re$ = Reynold's number.

First calculate $Re$, $Re = \frac{\rho V d}{\mu} = \frac{Vd}{(\frac{\mu}{\rho})} = \frac{Vd}{\nu} = \frac{3 \times 0.30}{0.01 \times 10^{-4}}$

$= 9 \times 10^5$

the value of $Re$ lies in the range of $4000 < Re$

So, it is a turbulent flow.

So, Value of $f = \frac{0.079}{Re^{1/4}} = \frac{0.079}{(9 \times 10^5)^{1/4}} = 0.00256$

Hence, head lost ($h_f$) $= \frac{4fLV^2}{2dg} = \frac{4 \times 0.00256 \times 50 \times 3^2}{2 \times 0.30 \times 9.81} = 0.7828$ m Ans

(ii) By chezy's formula

$V = C\sqrt{mi}$, where, $C = 60$

$m = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = d/4 = \frac{0.30}{4}$

$= 0.075$ m

$\therefore$ $3 = 60\sqrt{0.075 \times i}$

$\Rightarrow i = \left(\frac{3}{60}\right)^2 \times \frac{1}{0.075} = 0.0333$

Now, $i$ = ~~chezy's~~ loss of head per unit length of pipe.

$= \frac{h_f}{L}$

$\Rightarrow 0.0333 = \frac{h_f}{L} \Rightarrow h_f = L \times 0.0333$

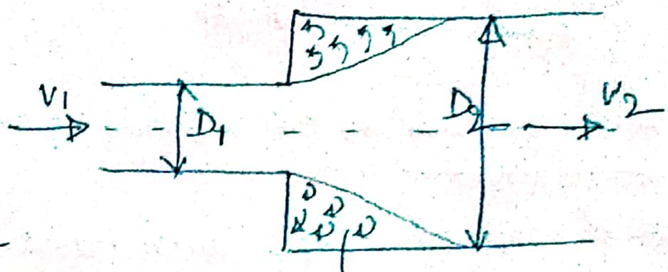
$= 50 \times 0.0333 = \boxed{1.665 \text{ m.}}$ Ans

# Minor energy (Head) losses

The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the flowing fluid in magnitude or direction is called minor loss of energy. The minor loss of energy (or head) includes the following cases:

## 1. Loss of head due to sudden enlargement :-

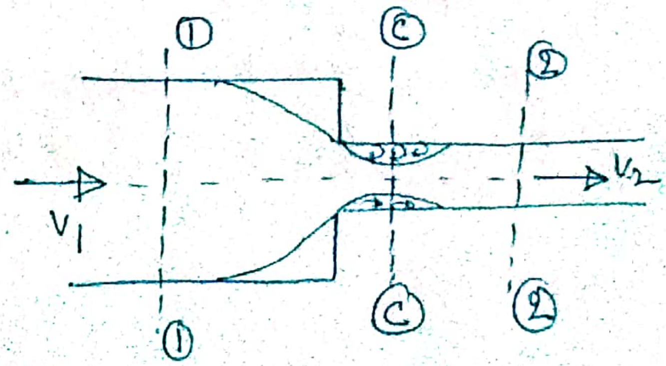
Due to sudden change of diameter of pipe from $D_1$ to $D_2$, the liquid flowing from the smaller pipe is not able to follow the abrupt change of the boundary. Thus the flow separates from the boundary and turbulent eddies are formed. The loss of head takes place due to the formation of these eddies.




$$\boxed{\text{loss of head } (h_e) = \frac{(V_1 - V_2)^2}{2g}}$$

## 2. Loss of head due to sudden contraction :-

As the liquid flows from large pipe to smaller pipe, the area of flow goes on decreasing and becomes minimum at section ©-©. This section ©-© is called Vena-contracta. After section



© - © a sudden enlargement of the area takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from vena-contracta to smaller pipe.

$$\boxed{\text{Loss of head } (h_c) = \frac{V_2^2}{2g}\left[\frac{1}{C_c}-1\right]^2 = K\frac{V_2^2}{2g},}$$ where, $K = \left[\frac{1}{C_c}-1\right]^2$

$$\boxed{C_c = \frac{A_c}{A_2}}$$

- If the value of $C_c$ is not given, then,

$$\boxed{h_c = 0.5\frac{V_2^2}{2g}}$$

## 3 Loss of head at the entrance of pipe -

This is loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir. This loss is similar to the loss of head due to sudden contraction.

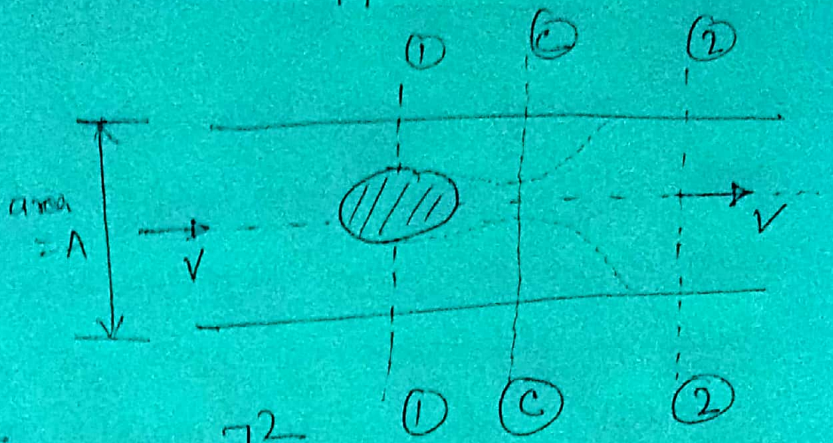
$$\boxed{h_i = 0.5\frac{V^2}{2g}}$$ , where $V$ = velocity of liquid in pipe

## 4 Loss of head at exit of a pipe -

This is the loss of head (or energy) due to the velocity of liquid at outlet of the pipe which is dissipated either in the form of a free jet or it is lost in the tank or reservoir.

$$\boxed{h_o = \frac{V^2}{2g}}$$ , where, $V$ is the velocity of liquid at the outlet of pipe.

## Loss of head due to an obstruction in a pipe :




$$\begin{pmatrix}\text{Loss of head due to} \\ \text{sudden obstruction}\end{pmatrix} = \frac{V^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]^2$$

where, $C_c$ = co-efficient of contraction

$= \frac{\text{area of vena-contracta}}{(A-a)}$

$= \frac{a_c}{(A-a)}$

## 6 Loss of head due to bend in pipe -

When there is any bend in a pipe, the velocity of flow changes, due to which the separation of flow from the boundary and also formation of eddies, takes place.

$$\boxed{h_b = \frac{kV^2}{2g}}$$ where, K = co-efficient of bend.

## 7 Loss of head in various pipe fittings -

The loss of head in various pipe fittings such as valves, couplings etc. is expressed as -

$$\boxed{h_f = \frac{kV^2}{2g}}$$ where, K = Co-efficient of pipe fitting

## * Hydraulic gradient line

- It is defined as the line which gives the sum of pressure head $\left(\frac{p}{\rho g}\right)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line (or) it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head $\left(\frac{p}{\rho g}\right)$ of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L.

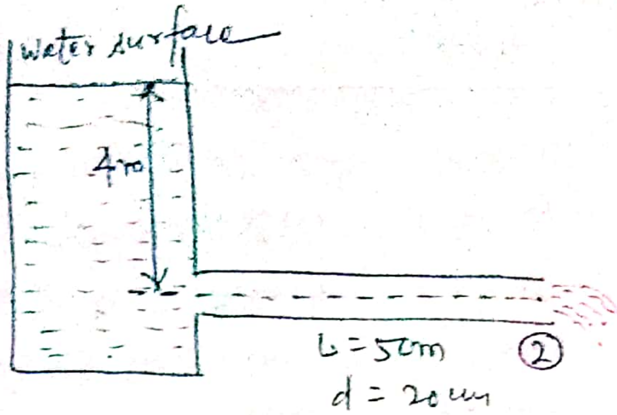
## * Total energy line

- It is defined as the line which gives the sum of pressure head $\left(\frac{p}{\rho g}\right)$, datum head (z) and kinetic head $\left(\frac{v^2}{2g}\right)$ of a flowing fluid in a pipe with respect to some reference line.
- It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as T.E.L.

Ex. Determine the rate of flow of water through a pipe of diameter 20cm and length 50m when one end of the pipe is connected to a tank and other end of the pipe is open to atmosphere. The pipe is horizontal and the height of water in the tank is 4m above the centre of pipe. Consider all minor losses and take $f = 0.009$ in the formule $h_f = \frac{4fLv^2}{2dg}$.

Also draw Hydraulic gradient line (H.G.L.) and Total energy line (T.E.L.)

dia of pipe (d) = 0.20m
Length of pipe (L) = 50m
Height of water (H) = 4 m
co-efficient of friction (f) = 0.009




let velocity of water in pipe = V m/s

Applying Bernoulli's equation at the top of water surface in the tank and at the outlet of pipe.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \text{all losses}$$

Considering datum line passing through the centre of pipe

$$0 + 0 + 4 = 0 + \frac{V_2^2}{2g} + 0 + (h_i + h_f)$$

$$\Rightarrow \quad 4 = \frac{V_2^2}{2g} + h_i + h_f$$

But velocity in pipe = V, $\therefore V = V_2$

$$4.0 = \frac{V^2}{2g} + h_i + h_f \quad \text{———} \; ①$$

$h_i$ = loss at the entrance $= 0.5\frac{V^2}{2g}$

$h_f$ = frictional loss in pipe $= \frac{4fLV^2}{2dg}$

substituting these value in eqn ①.

$$4 = \frac{V^2}{2g} + 0.5\frac{V^2}{2g} + \frac{4 \times 0.009 \times 50 \times V^2}{2g \times 0.2}$$

$$4 = \frac{V^2}{2g}\left[1 + 0.5 + \frac{4 \times 0.009 \times 50}{0.2}\right] = \frac{V^2}{2g}[1.5 + 9] = \frac{V^2}{2g} \times 10.5$$

$$\Rightarrow V = \sqrt{\frac{4 \times 2 \times 9.81}{10.5}} = 2.734 \text{ m/s}$$

Now, Rate of flow (or) Discharge $= A \times V = \frac{\pi}{4} \times (0.2)^2 \times 2.734 = 0.08589 \text{ m}^3/\text{s}$

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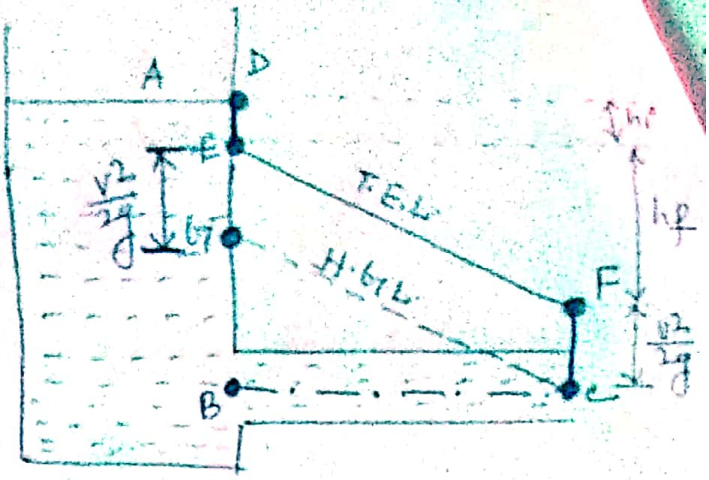
## Total energy line: (TEL)




Consider three points A, B and C on the free surface of water in the tank, at the inlet of the pipe and at the outlet of the pipe respectively. Now find total energy at these points, taking the centre of pipe as reference line.

(a) Total energy at A

$$= \frac{p}{\rho g} + \frac{v^2}{2g} + z = 0 + 0 + 4 = 4\,m$$

(b) Total energy at B = (Total energy at A) − (hi)

$$= 4 - 0.19 = 3.81\,m$$

(c) Total energy at C $= \frac{p_c}{\rho g} + \frac{v_c^2}{2g} + z_c$

$$= 0 + \frac{v^2}{2g} + 0 = \frac{(2.734)^2}{2 \times 9.81} = 0.38\,m.$$

hi = head lost at entrance of pipe

$$= 0.5\,\frac{v^2}{2g} = 0.5 \times \frac{2.734^2}{2 \times 9.81} = \boxed{0.19\,m}$$

hf = head lost due to friction in pipe

$$= \frac{4fLv^2}{2g \times d} = \frac{4 \times 0.009 \times 50 \times (2.734)^2}{2 \times 9.81 \times 0.2}$$

$$= \boxed{3.428\,m}$$

- Hence T.E.L will coincide with free surface of water in the tank. At the inlet of water it will decrease by $h_i = 0.19\,m$ from free surface and at the outlet of pipe total energy is 0.38 m.
- Point D represents total energy at A
- Point E " " " " inlet of the pipe. DE = hi = entrance loss.
- Point F " " " " outlet of the pipe. CF = 0.38 = total energy at the outlet.
- So, DEF represents the total energy line.

## Hydraulic gradient line :-

H.G.L gives the sum of $\left(\frac{p}{\rho g} + z\right)$ with reference to datum line. Hence H.G.L is obtained by subtracting $\left(\frac{v^2}{2g} = \text{Kinetic head}\right)$ from total energy. At outlet of the pipe, total energy $= \frac{v^2}{2g}$. By subtracting $\frac{v^2}{2g}$ from total energy at this point, we shall get point C, which lies on the centre line of the pipe. Draw a line CG parallel to EF. Then CG represents H.G.L.

- HGL is always parallel and lower than TEL.

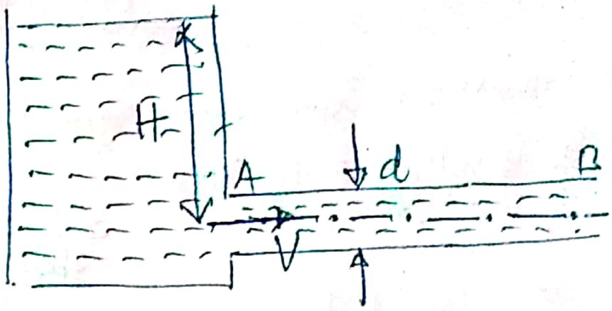
# Hydraulic Power transmission through pipes

Power is transmitted through pipes by flowing water or other liquids flowing through them. The power transmitted depends upon —
(I) the weight of the liquid flowing through the pipe, and
(II) the total head available at the end of pipe.

$$\boxed{\text{Power} = (\rho g Q) \times (\text{Head available at the end of pipe})}$$

Consider a pipe AB connected to a tank as shown —

Let. L = length of pipe
d = diameter of pipe
H = Total head available at the inlet of pipe
V = velocity of flow in pipe
hf = loss of head due to friction




If minor losses are neglected, then
(Total head available at the outlet of pipe) = (Total head at inlet) − (loss of head due to friction)

$$= H - h_f = H - \frac{4fLV^2}{2gd}$$

{f = coefficient of friction}

Weight of water flowing through pipe per second,

$$W = \rho g \times \text{volume of water per second}$$
$$= \rho g \times \text{Area} \times \text{velocity}$$
$$= \rho g \times \frac{\pi}{4} d^2 \times V$$

∴ Power transmitted at the outlet of pipe (P) =
(Weight of water per second) × (head at outlet)

$$\boxed{P = \left(\rho g \times \frac{\pi}{4} d^2 \times V\right) \times \left(H - \frac{4fLV^2}{2dg}\right) \text{ watts}} \quad \text{——— (1)}$$

or. Power transmitted at outlet of the pipe (in Kilowatt)

$$\boxed{P = \frac{\rho g}{1000} \times \frac{\pi}{4} d^2 \times V \left(H - \frac{4fLV^2}{2gd}\right)} \text{ KW} \quad \text{(II)}$$

Efficiency of power transmission,

$$\eta = \frac{\text{Power available at the outlet of pipe}}{\text{power supplied at the inlet of the pipe}}$$

$$= \frac{W \times (H - h_f)}{W \times H} = \frac{\rho g Q (H - h_f)}{\rho g Q\, H} = \frac{H - h_f}{H} \quad \text{(III)}$$

- Condition for maximum transmission of power:

For maximum power transmission, differentiate eqn (II) w.r.t. velocity (V) and equating it to zero.

$$\frac{dP}{dV} = 0$$

or $$\frac{d}{dV}\left[\frac{\rho g}{1000} \times \frac{\pi}{4} d^2 \left(HV - \frac{4fLV^3}{d \times 2g}\right)\right] = 0$$

or, $$\frac{\rho g}{1000} \times \frac{\pi}{4} d^2 \left(H - \frac{4 \times 3 \times f \times L \times V^2}{d \times 2g}\right) = 0$$

or $$H - 3 \times \frac{4fLV^2}{d \times 2g} = 0 \Rightarrow H - 3h_f = 0$$

$$\Rightarrow \boxed{h_f = \frac{H}{3}} \quad \text{***} \quad \text{(IV)}$$

Hence, power transmitted through a pipe is maximum when the loss of head due to friction ($h_f$) is one-third of the total head at inlet (H).

- Maximum efficiency of transmission of power:

We know that condition for max$^m$ power transfer, $\boxed{h_f = \frac{H}{3}}$

Substituting the value of $h_f$ in efficiency, we get maximum efficiency.

$$\eta_{max} = \left(\frac{H - h_f}{H}\right)_{h_f = H/3} = \frac{H - H/3}{H} = 1 - \frac{1}{3} = \frac{2}{3} = 66.7\%$$



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